

Renormalization Group independence of Cosmological Attractors

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The large class of inflationary models known as α - and ξ -attractors give identical predictions at tree level (at leading order in inverse power of the number of efolds). Working with the renormalization group improved action, we show that these predictions are robust under quantum corrections. This result follows once the field dependence of the renormalization scale, fixed by demanding the leading log correction to vanish, satisfies a quite generic condition. In Higgs inflation this is indeed the case; in the more general attractor models this is still ensured by the renormalizability of the theory in the effective field theory sense.

I. INTRODUCTION

Successful inflationary models should satisfy some basic requirements. They have to be consistent within the theory in which they are formulated (QFT and GR). Moreover, for the models to be predictive, the predictions should depend on a number of parameters smaller than the number of predictions themselves.

A large class of inflationary models, the so-called Cosmological Attractors, give the same classical predictions for the inflationary observables¹ [1]. At leading order in the $1/N$ expansion, with N the number of efolds, the tree-level spectral index and tensor-to-scalar ratio are given by $n_s = 1 - 2/N + O(N^{-2})$ and $r = 12/N^2 + O(N^{-3})$. A natural question to ask then is if these predictions are robust in the full quantum theory. Are the attractor models consistent and predictive at the quantum level? In this note we will consider the effect of perturbative corrections due to the renormalization group (RG) flow.

In single field inflation, the quantum corrected dynamics of the inflaton is given by an effective action of the form

$$\frac{\mathcal{L}_{\text{eff}}}{\sqrt{-g}} = -\frac{1}{2}Z(\phi)K_\phi(\phi)(\partial\phi)^2 - V(\phi) + \dots \quad (1)$$

with $Z(\phi)$ the (non-trivial) renormalization wavefunction, K_ϕ the metric in field space in presence of a non-canonical kinetic term, V the full quantum potential and the dots stand for higher derivative terms that can be safely neglected in the slow roll approximation. With the gravity sector in the standard form, i.e. for the action in the Einstein frame, the slow roll

parameters as well as the inflationary indexes n_s and r are given in terms of derivatives of the effective potential with respect to the canonical inflaton field. Using standard renormalization group techniques it is possible to rewrite the effective action in a form suitable for the inflationary analysis, which takes into account the leading log expansion of the quantum potential.

We are interested in the possibility that quantum corrections enter at first order in the $1/N$ -expansion. The observables can then be written in the general form

$$n_s \simeq 1 - \frac{2}{N}f_n(\beta_{\lambda_i}, \lambda_j)_*, \quad r \simeq \frac{12}{N^2}f_r(\beta_{\lambda_i}, \lambda_j)_*, \quad (2)$$

evaluated at the time the pivot scale ($k_* = 0.002 \text{ Mpc}^{-1}$) leaves the horizon (denoted by a subscript $*$). Here f_n and f_r are two generic functions of the beta functions β_i and the couplings of the model λ_i . If the inflationary parameters have such a dependence,² this would imply that the knowledge of the details of the renormalization group (RG) flow during inflation are needed (to find out the expressions for β_{λ_i}) to draw conclusions on the model. Even more, to ever connect the low and high energy regimes of the model, one would need to know the details of the RG flow through the entire energy domain.

The RG dependence of the observables can be both a curse and a blessing. A blessing because it can lift the degeneracy between the different attractor models. Moreover, including loop corrections to the inflaton action could in principle shed light on the UV dependence of the inflationary parameters. On the other hand, if a model depends strongly on unknown UV corrections it will loose completely any predictive power.

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¹ More precisely, for $\alpha \rightarrow 1$ and $\xi \rightarrow \infty$ in the α - and ξ -attractors respectively.

² Note that even though β_{λ_i} might be small in general, this is not necessarily true for combinations of $(\beta_{\lambda_i}, \lambda_j)$, e.g. $\beta_{\lambda_i}/\lambda_i$.

In this note, generalizing the idea of our previous work [2], we show that for the large class of inflationary attractor models, the α - and ξ -attractors [3–6], n_s and r are nearly independent on the RG flow. This also implies that any kind of UV physics whose effect enters only via the RG flow (i.e., that does not affect the inflationary potential already at tree level) will in general have no effect on the predictions for these models.

A. RG improving and renormalization scale

Let us briefly review some standard features of the effective action and the RG flow that we will use in the following. The quantum potential for a scalar field ϕ depends in general on powers of logarithms of the form $\ln(M_i^2(\phi)/\mu^2)$ where $M_i^2(\phi)$ are the field-dependent masses of the particles running in the loops and μ the renormalization scale. The logarithms appear only up to L -th power at the L loop order. A well known result in quantum field theory [7, 8] tells us that in each region of the field space it is possible to define an effective field theory (EFT) where only one logarithm remains relevant in the full effective potential. All the other mass scales decouple and their net effect will be a shift in the definition of the parameters of the EFT. Thus, schematically, each loop contribution will have the following form

$$V^{(L)} = \hbar^{-1}(v_0^{(L)} s^L + \hbar v_1^{(L)} s^{L-1} + \dots + \hbar^L v_L^{(L)}), \quad (3)$$

where

$$s = \hbar \ln \left(\frac{M^2(\phi)}{\mu^2} \right) \quad (4)$$

is the only relevant log in the EFT. Here $v_i^{(j)}$ are functions of the field and all the other couplings/mass parameters (λ_i), and \hbar is the loop counting parameter. The full potential can be written in general as [9]

$$\begin{aligned} V &= M^4(\phi) \sum_{i=0}^{\infty} \hbar^{i-1} \left(\sum_{L=i}^{\infty} v_i^{(L)} s^{L-i} \right) \\ &\equiv M^4(\phi) \sum_{i=0}^{\infty} \hbar^{i-1} f_i, \end{aligned} \quad (5)$$

where f_i is the i -th to leading log term. We label the potential truncated at L -loop order with $V_L = V^{(0)} + \dots + V^{(L)}$. V satisfies the renormalization group equation (RGE) [10, 11]:

$$DV \equiv \left(\mu \frac{\partial}{\partial \mu} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} - \gamma \phi \frac{\partial}{\partial \phi} \right) V = 0, \quad (6)$$

where γ is the anomalous dimension of the scalar field. This allows us to rewrite it as a formal solution in the following standard way (see for example [12])

$$V(\phi, \lambda_i, \mu) = V(\bar{\phi}(t), \bar{\lambda}_i(t), \bar{\mu}(t)) \quad (7)$$

where

$$\begin{aligned} \frac{d\bar{\phi}(t)}{dt} &= -\gamma(\bar{\lambda}_i(t))\bar{\phi}(t), \quad \frac{d\bar{\lambda}_i(t)}{dt} = \beta_i(\bar{\lambda}_j(t)), \\ \bar{\mu}(t) &= \mu e^t, \end{aligned} \quad (8)$$

and with the initial conditions that the barred quantities reduce to the unbarred ones at $t = 0$. In a standard quantum field theory the renormalizability ensures that in each EFT the RG operators \mathcal{D} are the same. Indeed, we are actually solving the same equation by simply using different set of parameters. The matching between the solutions is provided by the equations that relate the parameters of two adjacent EFTs evaluated at the renormalization point μ around the threshold [8].

Given a generic inflationary model renormalizable in the EFT sense in the inflationary regime,³ the operators \mathcal{D} are not necessarily the same in each EFT (defined at different energy scales). To patch together the EFTs we would then need some threshold corrections, i.e. some extra UV physics [2, 13, 14]. This is for example the case in Higgs inflation [15, 16] where the beta functions are different in the low and middle/large regime [17–22]. The UV physics can be parameterized by a tower of higher order operators, which have a net effect on the boundaries of the EFTs and as such provide the necessary threshold corrections [23]. In this way the UV physics can enter the predictions through the RG flow, that is different Wilson coefficients for the higher order operators could result in different n_s and r . As we will show in this paper, it turns out that for a large class of inflationary models we do not really need to know the details of the RG flow to derive the inflationary parameters.

When we formally solve the RG equation in the inflationary regime, only one log remains relevant. Equation (7) tells us that the effective potential is determined once its functional form is known for a certain value of t . The standard procedure to derive useful information from (7), is to choose t in such a way that

$$\bar{s}(\tilde{t}) = \ln \frac{\bar{M}^2(\tilde{t})}{\bar{\mu}^2(\tilde{t})} = 0 \implies \tilde{t}(\phi, \lambda_i(\tilde{t})) \quad (9)$$

³ In each field region it is possible to define a small parameter; there should be a finite number of counter terms at every order in the expansion in this small parameter.

where we omit from now on the bar over the running couplings. In this way $V = V(t)|_{\bar{s}=0} = V_L(t)|_{\bar{s}=0} + O(\hbar^L)$.⁴ This means that the knowledge of the L loop potential (and the function \tilde{t}) provides an exact RG improved potential up to order L in this leading log expansion. Therefore, depending on the order we want to work at, the potential used in our computations is given by

$$V_L|_{\bar{s}=0} \equiv V_L(\tilde{t}), \quad (10)$$

and the RGE coefficients functions β, γ at $(L+1)$ -loop order [7, 8]. We will consider the leading corrections, that is we set $L = 0$ and use the 1-loop β -functions. Even if our results will not depend on the loop order of the β -functions considered, this does not imply that it holds automatically beyond the leading order. In section IID we comment on the generalization of our results to higher orders.

Let us make an important remark here. In the following (we used this already in (9)) we will consider ϕ instead of its barred and t dependent version $\bar{\phi}(t)$ in the RG improved potential (and ρ instead of $\bar{\rho}(t)$ in the next sections). We are allowed to do this for the following reason (see [24] and our appendix A in [2] for more details). Consider the improved renormalization wavefunction as absorbed in the field redefinition, i.e. $Z_{\text{eff}}(t)(\partial\phi)^2 = (\partial\phi_{\text{can}})^2$. Then we have, at leading order

$$\bar{\phi} = e^{-\int \gamma dt'} \phi = e^{-\int \gamma dt'} Z_{\text{eff}}^{-1/2} \phi_{\text{can}} \approx \phi_{\text{can}}, \quad (11)$$

where we simply omit the subscript “can”.

B. Key idea

The key point is that since only one log remains relevant during inflation there is (up to some irrelevant numerical factors) a unique choice for the function \tilde{t} , i.e. the one implicitly defined by eq. (9). In order to compute the inflationary parameters n_s and r we take derivatives of the effective potential with respect to the scalar field. These will be a function of derivatives of \tilde{t} as well as of the couplings and the β -functions. Thus the predictions can in principle depend on the RG flow during inflation (through the value of the beta functions in this regime) and on the full RG flow (through the value of the running couplings at the horizon).

Expanding the equations in powers of the small parameter ρ defining the inflationary regime it can be shown analytically, without having to solve explicitly the RG equations, that for the inflationary Cosmological Attractors models, neither of the two contributions influence the inflationary predictions at first order in inverse power of the number of e-folds.⁵

II. INFLATIONARY PARAMETERS

A. General set up: tree level

Let us start by reviewing the predictions for the Cosmological Attractors at tree level [1, 3–6]. The Lagrangian of the models considered can be written as (with the Planck mass set to one)

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} K(\rho)(\partial\rho)^2 - V(\rho) \right], \quad (12)$$

with

$$K = \frac{a_p}{\rho^p} + \frac{a_{p-1}}{\rho^{p-1}} + \dots, \quad V = V_0(1 + c\rho + c_2\rho^2 + \dots) \quad (13)$$

where $\rho \ll 1$ is the parameter identifying the inflationary regime. V is the tree level potential in the Einstein frame (in the previous section labeled with $V^{(0)}$), while $V_0 = V|_{\rho=0}$ is the coupling dependent part of it.⁶

To first approximation the following happens [1]: the slow roll parameter η , and consequently the spectral index, is completely determined by the order of the leading pole in the kinetic term (p); for $p = 2$ the tensor-to-scalar ratio r will depend only on the residue of this leading pole.⁷ We will now show this explicitly.

The first and second slow roll parameters are

$$\epsilon = \frac{1}{2} \left(\frac{V_\rho}{V} \right)^2 K^{-1} = \frac{1}{2} \left(\frac{V_\rho}{V} \right)^2 \frac{\rho^p}{a_p}, \quad (14)$$

and

$$\begin{aligned} \eta &= \frac{V_{\chi\chi}}{V} = -\frac{V_\rho}{V} \frac{d^2\chi}{d\rho^2} \left(\frac{d\chi}{d\rho} \right)^{-3} + \frac{V_{\rho\rho}}{V} \left(\frac{d\chi}{d\rho} \right)^{-2} \\ &= \frac{p}{2} \frac{\rho^{p-1}}{a_p} c + O(\rho^p). \end{aligned} \quad (15)$$

⁵ The scalar power spectrum constrains one combination of couplings in the theory, which can always be satisfied fixing the free parameter of the model. For different RG evolutions, the individual couplings may have different values at horizon exit, but as long as the combination is kept fixed by adjusting the free parameter, this has no direct observable consequences.

⁶ The reason for these choices is to adopt the same notation as [1].

⁷ This approach is robust under perturbation of the non-canonical kinetic term K with terms of one order higher in the leading pole, i.e. $K \subset a_{p+1}/\rho^{p+1}$ [25].

⁴ It might seem obvious that V and V_L differ by $O(\hbar^L)$ terms. However, what we mean by that are order \hbar^L terms in the leading log series expansion (5).

Here χ labels the canonical field defined via $d\chi/d\rho = K^{\frac{1}{2}}$. Further we introduced the notation $V_\rho = dV/d\rho$, and likewise for higher derivatives. From the previous expression we see that in all these models $\eta \gg \epsilon$ (one order in ρ difference). The number of efolds is given by

$$N \simeq \int^{\rho_\star} \left| \left(\frac{V}{V_\rho} \right) \right| K |d\rho| \simeq \frac{a_p}{|c| \rho_\star^{p-1} (p-1)}, \quad (16)$$

which implies

$$\rho_\star = \left[\frac{N|c|(p-1)}{a_p} \right]^{\frac{1}{1-p}}. \quad (17)$$

Evaluating the slow roll parameters at horizon exit then gives

$$\begin{aligned} \eta_\star &\simeq \frac{p}{2(p-1)} \frac{1}{N} \text{sign}[c], \\ \epsilon_\star &\simeq \frac{1}{2} |c|^{\frac{p-2}{p-1}} a_p^{\frac{1}{p-1}} \left(\frac{1}{(p-1)N} \right)^{\frac{p}{p-1}}. \end{aligned} \quad (18)$$

For $p = 2$ all dependence on the potential drops from the inflationary parameters (apart from the spurious dependence on the sign of c), which become

$$n_s \simeq 1 + \text{sign}[c] \frac{2}{N}, \quad r \simeq \frac{8a_2}{N^2}. \quad (19)$$

In all the models considered the sign of c is negative (concave potential) and the potential in terms of the canonical field is characterized by a flat plateau. The spectral index is then given by $n_s \simeq 1 - \frac{2}{N}$.

B. General set up: Quantum corrections

As we discussed in section I A, even if the inflaton field has a non-canonical kinetic term, the net effect of considering leading order quantum corrections is captured by substituting in the tree level action each coupling by its running counterpart, i.e. $\lambda_i \rightarrow \lambda_i(t)$, modulo a proper choice of the RG time $t = \tilde{t}$ (the one solving (9)). Thus, we consider the RG improved version of (12), given by

$$\frac{\mathcal{L}^{\text{eff}}}{\sqrt{-g}} \simeq -\frac{1}{2} K(\rho, \lambda_i(\tilde{t}(\rho))) (\partial\rho)^2 - V(\rho, \lambda_i(\tilde{t}(\rho))). \quad (20)$$

For a potential of the form (13), the dependence on the couplings is in V_0 . The derivative of the potential

with respect to ρ , denoted by V_ρ , then becomes⁸

$$V_\rho = V_0 \sum_{n=1}^{\infty} n c_n \rho^{n-1} + \frac{\beta_{V_0}}{V_0} \frac{d\tilde{t}}{d\rho} V \quad (21)$$

where $c_1 \equiv c$ (to match the tree level notation (13)). At leading order we have

$$\frac{V_\rho}{V} \simeq c + \frac{\beta_{V_0}}{V_0} \frac{d\tilde{t}}{d\rho}. \quad (22)$$

Now a key point of the argument kicks in. On the weak assumption that \tilde{t} can be simply expanded in a Taylor series about zero,

$$\frac{d\tilde{t}}{d\rho} = \sum_{k=0}^{\infty} d_k \rho^k, \quad (23)$$

where the coefficients d_k can depend implicitly on ρ , we have

$$\frac{V_\rho}{V} = c + \frac{\beta_{V_0}}{V_0} d_0 + O(\rho). \quad (24)$$

Thus, the effect of the RG flow will be only a rescaling of the factor

$$c \rightarrow \mathcal{C} \equiv c \left(1 + \frac{\beta_{V_0}}{V_0} \frac{d_0}{c} \right). \quad (25)$$

As we show now, this will be the only relevant effect which has no consequences for the inflationary parameters.

Let us compute the number of efolds. The leading term in the integrand is the same as (16) with the replacement (25). On the other hand, the factor $a_2/|\mathcal{C}| \equiv D$ is not a constant anymore and it depends implicitly on ρ . Expanding it in a Taylor series about ρ_\star gives

$$\begin{aligned} N &\simeq \int^{\rho_\star} \frac{|d\rho|}{\rho^2} \frac{a_2}{|\mathcal{C}|} \\ &= \int_{\rho_\star} \frac{d\rho}{\rho^2} \left(D_\star + \beta_{D_\star} \frac{d\tilde{t}}{d\rho} (\rho - \rho_\star) + \dots \right). \end{aligned} \quad (26)$$

Given the assumption (23), we observe that D can be considered constant over the integration domain within our approximation, i.e. $D \simeq D_\star$. In fact, all the other terms, starting from the second one within the brackets, give contributions that are at most of order

⁸ Note that for the set-ups considered, even if the coefficients c_i had a dependence on the couplings that would only give contributions to the derivative that are higher order in ρ , of the form $\sim V_0 \left(\rho \frac{dc}{d\rho} \frac{d\tilde{t}}{d\rho} + \dots \right)$.

$\sim \ln \rho_*$, which gives an order higher in the $1/N$ expansion. These contributions enter at the same order as the corrections from the subleading poles in the kinetic term, which were already neglected at tree level. The number of efolds then becomes

$$N \simeq \frac{a_{p*}}{|\mathcal{C}_*| \rho_*}, \quad (27)$$

which is simply (16) with $p = 2$ and the couplings (which now are not constant anymore) evaluated at the horizon crossing ρ_* . Therefore ϵ_* will be exactly the same as the tree level expression, but with c replaced by \mathcal{C}_* (whose dependence drops out for $p = 2$). In computing η , one has to be a little more careful since an extra contribution could come from the derivative of the non-canonical kinetic term in (20). In fact, in (15), we should consider

$$\frac{dK^{\frac{1}{2}}}{d\rho} \equiv \frac{d^2\chi}{d\rho^2} = \frac{1}{2}\rho^{\frac{p}{2}}a_p^{\frac{1}{2}} \left(-p\rho^{-p-1} + \frac{\beta_{a_p}}{a_p} \frac{d\tilde{t}}{d\rho} \rho^{-p} \right) \Big|_{p=2}. \quad (28)$$

However, as long as (23) is satisfied, the second term in this expression gives higher order contributions to η . Thus (15) becomes

$$\eta = -\frac{V_\rho}{V} \frac{d^2\chi}{d\rho^2} \left(\frac{d\chi}{d\rho} \right)^{-3} + O(\rho^2) = \rho \frac{\mathcal{C}}{a_2} + O(\rho^2), \quad (29)$$

which is the tree level expression at leading order with again \mathcal{C} playing the role of c . It is then obvious that inverting (27) and substituting ρ_* in the the slow roll parameters gives the same (19) for n_s and r ,

$$n_s \simeq 1 + \text{sign}[\mathcal{C}_*] \frac{2}{N}, \quad r \simeq \frac{8a_{2*}}{N^2}. \quad (30)$$

Summarizing, the effect of the RG flow enters in three ways. First, in the efolds dependence of $\rho_* = \rho_*(N)$. Second, by giving a rescaling $c \rightarrow \mathcal{C}$ in the slow roll parameters (before evaluating them at the horizon crossing) and third from the extra contribution to the derivative of K in η . Nevertheless, if the condition (23) is satisfied, this latter gives simply higher order contributions, while the first and second points compensate each other. Different running histories (encoded in $(\beta_{V_0}/V_0)_*$ in \mathcal{C}_*) will just imply a different value of the field at the horizon exit ρ_* . This effect cancels with the shifted expressions of the slow roll parameters.⁹

⁹ Suppose that (23) is not satisfied, for example $\tilde{t} \propto \frac{k}{\rho}$. In the number of efolds the second term in (26) will now give $\int \frac{d\rho}{\rho^2} \beta_{D*} \frac{d\tilde{t}}{d\rho} (\rho - \rho_*) = \frac{k}{\rho_*} \beta_{D*} + \text{h.o.}$, which is of the same order as the leading term D_*/ρ_* , and thus (27) and the relation $\rho_*(N)$ is altered. As a consequence, the leading order slow roll parameters at horizon exit will depend on the beta functions.

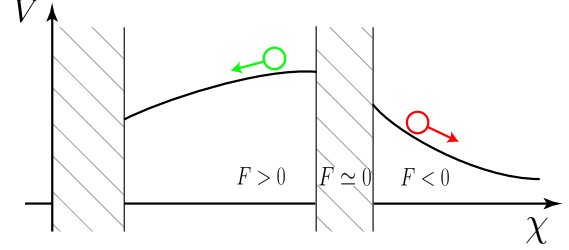


Figure 1. Possible features of the effective potential due to RG effects.

Note that the arguments presented are valid as long as the quantum corrections encoded in \mathcal{C} do not break the perturbative expansion in ρ . In general $|\mathcal{C}| \sim O(1)$ and (30) follows. Consider the term between brackets in (25), denoted by F , i.e.

$$\mathcal{C} = c \left(1 + \frac{d_0}{c} \frac{\beta_{V_0}}{V_0} \right) \equiv cF, \quad (31)$$

if F is positive during inflation, then $\text{sign}[\mathcal{C}_*] < 0$ and the predictions will be the same as for the tree level case.

C. Maximum and breakdown of perturbativity

The conclusion that the tree level predictions are not affected by the RG corrections is valid as long as the perturbative expansion in ρ holds and inflation takes place on the plateau of the potential. It may be that the potential develops an extremum because of the running; this is purely a quantum effect in that the tree level potential has no extremum in the inflationary regime. For fine-tuned parameters it is then possible to obtain inflation near the maximum or inflection point. In this case the details of the inflationary scenario will depend sensitively on the quantum corrections.

To see the appearance of an extremum in the potential, consider its slope. For $|F| \gtrsim \rho$ the perturbative expansion is valid and $V_\rho = V_0 c F + (\rho)$, see (24). It follows that going from a region in field space with $F \gtrsim \rho$ to a region with $F \lesssim -\rho$, the slope of the potential has changed sign. This can only happen if there is a (at least one) maximum in between. In general, one cannot calculate the location of the maximum analytically though, as the perturbative expansion breaks down exactly in the in between region where $F \sim 0$.

For the particular choice of normalization scale $\bar{\mu}(\tilde{t}) \propto V^{\frac{1}{4}}$ the slope of the potential factorizes in a classical piece times a quantum correction at all orders, see (34) below. Then it can actually be shown analytically that the regime where perturbativity breaks down

coincides with the development of a maximum in the effective potential. This choice of normalization scale is appropriate for Higgs inflation and it also appears generically in the α and ξ -attractor models considered. It can be parametrized $\bar{\mu}(\tilde{t}) = (V_0\mu_0)^{1/4}(1+c\rho+..)^{1/4}$, where V_0 and μ_0 depend explicitly on the couplings but do not explicitly depend on the field ρ . It is not hard to see that this satisfies (23). The RG time is then

$$\tilde{t} = \ln \bar{\mu}(\tilde{t}) = \frac{1}{4} \ln(\mu_0 V). \quad (32)$$

Taking the derivative with respect to ρ gives

$$\frac{d\tilde{t}}{d\rho} = \frac{1}{4} \left(\frac{V_\rho}{V} + \frac{\beta_{\mu_0}}{\mu_0} \frac{d\tilde{t}}{d\rho} \right) \implies \frac{d\tilde{t}}{d\rho} = \frac{V_\rho/4V}{1 - \beta_{\mu_0}/4\mu_0}, \quad (33)$$

which allows us to write an exact expressions for V_ρ without needing to solve (21) iteratively. Inserting the previous expression in (21) gives

$$V_\rho = V_0 \left(\sum_{n=1}^{\infty} n c_n \rho^{n-1} \right) \frac{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0} \right)}{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0} - \frac{\beta_{V_0}}{4V_0} \right)}, \quad (34)$$

and thus¹⁰

$$\mathcal{C} = c \frac{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0} \right)}{\left(1 - \frac{\beta_{\mu_0}}{4\mu_0} - \frac{\beta_{V_0}}{4V_0} \right)} \equiv cF. \quad (35)$$

Now switching to the canonical field χ , we thus find that for $F = 0$ the slope vanishes $V_\chi = -K^{-\frac{1}{2}} F(c\rho + O(\rho^2)) = 0$.¹¹ The inflaton potential develops an extremum and ϵ is identically zero at all orders. Approaching this point, when $F \simeq \rho$, our perturbative analysis breaks down. To show that the potential develops a maximum consider the curvature at the extremum $V_{\chi\chi}|_{F=0} = K^{-1} F_\rho c(1 + O(\rho))$. As c is negative, it follows that $\text{sign}[V_{\chi\chi}]|_{\rho_{\text{ext}}} = -\text{sign}[F_\rho]|_{\rho_{\text{ext}}}$ where ρ_{ext} is the field value at the extremum. Now since $F(\rho) > 0$ for $\rho > \rho_{\text{ext}}$, we have $F_\rho(\rho_{\text{ext}}) \geq 0$. This implies that $\text{sign}[V_{\chi\chi}]|_{\rho_{\text{ext}}} < 0$, i.e. the extremum is indeed a maximum (or an inflection point for double fine-tuned parameters [26–28]). Note that, also this result is independent on the particular β -functions.

One can contemplate the possibility of inflation happening on the other side of the maximum. F is negative here as $\text{sign}[\mathcal{C}_*]$ is reversed, and thus $n_s > 1$.

¹⁰ Note that (31) and (35) are in agreement. Combining (33) and (34), $d\tilde{t}/d\rho$ can be written in the form (23) with $d_0 = (c/4)(1 - \frac{\beta_{\mu_0}}{4\mu_0} - \frac{\beta_{V_0}}{4V_0})^{-1}$. Now using (31) gives for F the same expression as in (35).

¹¹ Here we used that $d\rho/d\chi = -K^{-\frac{1}{2}} < 0$ since $K > 0$ and $\rho \rightarrow 0$ for $\chi \rightarrow \infty$.

However, this describes a completely different kind of inflation with the inflaton rolling on the other side of the maximum (see Fig. (1)). Therefore, when a maximum develops due to quantum corrections, one has to assume initial conditions such that inflation starts with the inflaton always on the “correct side” of the maximum. In Higgs inflation this is equivalent to the observational request to end up in the electroweak vacuum after inflation. For the wider class of models considered here, the assumption is still reasonable since the vacuum at the origin is tuned to have zero (small) cosmological constant and this is where inflation is assumed to end; the minimum at large field values might not only be large (negative or positive), but also in the regime where any calculational control is lost as quantum gravity corrections may be large.

D. Higher orders

Even if the arguments shown in section I A are general, it is still not clear if the result presented in section I B hold beyond leading order (LO). Consider the RG improved L -loops potential, i.e. $V = V_{0,\text{eff}}(\tilde{t})(1 + ..)$ where $V_{0,\text{eff}} = V_0(\tilde{t}) + V_0^1(\tilde{t}) + ..$ only depend explicitly on the couplings. Since the cancellation of the RG effects in the inflationary predictions does not depend explicitly on the particular V_0 nor on the order of β_{V_0} , everything still follows replacing $V_0 \rightarrow V_{0,\text{eff}}$ and $\beta_{V_0} \rightarrow \beta_{V_{0,\text{eff}}}$. However, including higher loops contributions we can in general no longer absorb the effect of the anomalous dimensions in the canonical field, as was discussed at the end of section I A. This may not necessarily hold beyond LO, for which further investigations would be required.

III. APPLICATIONS

We now discuss the classes of models that can be written in the general form (13); these are the α - and ξ - attractors. As a particular case of this latter we first consider Higgs inflation (HI) [15]. Here the condition (23) on the renormalization scale is determined by couplings of the Higgs to the other Standard Model particles. Nothing guarantees beforehand that this is still valid for the more general class of models considered. Nevertheless, in III D we discuss how the weak condition (23) holds naturally also for the Cosmological Attractors.

It thus follows that the tree level predictions for all these models are robust against quantum corrections.

A. Higgs Inflation

The argument outlined in the previous section applies to Higgs inflation. This is the reason why in [2] it was found that the dependence on the β -functions drops out of the inflationary predictions. In fact, the kinetic term and the potential for Higgs inflation (in the Einstein frame and in unitary gauge) can be written as a Laurent series in $\rho = \Omega^{-1} = 1/(1 + \xi\phi^2)$ as

$$K = \frac{3}{2\rho^2} + \frac{1}{4\xi(1-\rho)\rho^2} \simeq \frac{3}{2} \left(1 + \frac{1}{6\xi}\right) \frac{1}{\rho^2} + \frac{1}{4\xi} \sum_{i=1}^{\infty} \rho^i \quad (36)$$

and

$$V = V_0(1 - 2\rho + \rho^2) \quad (37)$$

with $V_0 = \lambda/(4\xi^2)$. This is exactly of the general form (13) with $p = 2$, $c = -2$ and $a_2 = 3/2(1 + 1/6\xi)$. The slow roll parameters at tree level are given by (18), which implies

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq 12 \left(1 + \frac{1}{6\xi}\right) \frac{1}{N^2}. \quad (38)$$

Large values of ξ are needed to fit the power spectrum of the scalar perturbations. In this limit the ξ -dependence disappears from the tensor-to-scalar ratio.

Let us now turn to the quantum corrections. In Higgs inflation there is a natural choice for the RG time \tilde{t} , which is chosen such that it minimizes the largest logs in the Coleman-Weinberg potential in agreement with (9). If Higgs inflation is embedded in the Standard Model, the dominant quantum corrections come from the W and Z bosons and from the top quark masses [17]. Their masses all scale the same way with the Higgs field, namely as $M_W = gf(\phi)/2$, $M_Z = (g^2 + g'^2)^{1/2}f(\phi)/2$, $M_t = y_t f(\phi)/\sqrt{2}$ with g_1, g_2, y_t the $U(1)$, $SU(2)$ and Yukawa couplings respectively and $f(\phi) = \phi/\Omega^{1/2}$. Using the notation of section I A, we choose for simplicity

$$s = \ln \left(\frac{f(\phi)}{\mu} \right). \quad (39)$$

The logs is minimized for $\bar{s}(\tilde{t}) = 0$ which implies

$$\tilde{t} = \ln \frac{\phi}{(1 + \xi(\tilde{t})\phi^2)^{1/2}} = \frac{1}{4} \ln \left(\frac{4V(\tilde{t})}{\lambda(\tilde{t})} \right). \quad (40)$$

To get the last expression it was used that the classical potential can be written as $V = \lambda f^4(\phi)/4$. We have given the masses and renormalization scale in the

Einstein frame, as this is where the inflationary observables are most easily computed. We note however that physics is frame independent; even if initially the loop corrections and renormalization scale is computed in the Jordan frame, and only afterwards the results are transformed to the Einstein frame, this would give the same result for the renormalization scale (40) [2, 29].

It follows that \tilde{t} is actually of the form (32) with $\mu_0 = 4/\lambda$ which satisfies the generic assumption (23). Using (33), (34) and (35) with $\beta_{\mu_0}/\mu_0 = -\beta_\lambda/\lambda$ and $\beta_{V_0}/V_0 = \beta_\lambda/\lambda - 2\beta_\xi/\xi$ gives¹²

$$\frac{d\tilde{t}}{d\rho} = -\frac{1}{2} \frac{1}{\left(1 + \frac{\beta_\xi}{2\xi}\right)} + O(\rho) \quad (41)$$

and

$$C = c \left(1 + \frac{\beta_{V_0}/4V_0}{1 - \frac{\beta_{\mu_0}}{4\mu_0}} \right) = -2 \left(\frac{1 + \frac{\beta_\lambda}{4\lambda}}{1 + \frac{\beta_\xi}{2\xi}} \right) \equiv -2F. \quad (42)$$

Since Higgs inflation is a specific example of the general set-up considered in section II B the shift of C by the quantum corrections drops out of the inflationary predictions, which are equal to the tree level results. This conclusion holds as long as the perturbative expansion in ρ is valid. As discussed in II C for $F_\star \simeq \rho_\star$ the potential develops a maximum, which for fine-tuned parameters can be used for ‘‘hilltop inflation’’ in agreement with the CMB data. This possibility was studied numerically in [2, 28].

Our choice of renormalization scale (40) is referred to as ‘‘prescription I’’, see [29] and section 2.3 of [2] for an extensive discussion. This is the natural choice since it minimizes the log preserving the asymptotic shift symmetry of the potential [2, 30]. In the literature ‘‘prescription II’’ has been also considered as well, defined by

$$\tilde{t} = \ln \phi = \frac{1}{2} \ln \left[\frac{1}{\xi} \left(\frac{1-\rho}{\rho} \right) \right]. \quad (43)$$

This corresponds to a different UV completion of the theory (where the potential is already altered at tree

¹² This can be matched to the notation used in [2], where the small parameter $\delta = (\xi\phi^2)^{-1}$ was used as expansion parameter. In that notation $\rho = \delta(\delta + 1)^{-1} \simeq \delta + O(\delta^2)$ and $\phi = \left(\frac{1-\rho}{\xi\rho}\right)^{1/2}$, which gives

$$\frac{dt}{d\phi} = \frac{\xi^{1/2} \delta^{3/2}}{1 + \frac{\beta_\xi}{2\xi} + \delta}, \quad \frac{d\phi}{d\rho} = -\frac{1}{2\xi^{1/2}} \frac{\rho^{-3/2}}{(1-\rho)^{1/2}} + \frac{\beta_\xi(1-\rho)^{1/2}}{\xi^{3/2} \rho^{1/2}} \frac{dt}{d\rho},$$

It then follows that at leading order $\frac{dt}{d\rho} = \frac{dt}{d\phi} \frac{d\phi}{d\rho}$ agrees with (41).

level in the large field regime). Using this prescription for the renormalization scale we can immediately see that the previous cancellation does not take place anymore. In fact $\frac{dt}{d\rho} = -\frac{1}{2} \frac{1}{\rho(1-\rho)}$ which is not of the form (23). This is the reason why in [14, 17, 31], where prescription II was considered, features for n_s and r have been observed.

B. α -attractors

The α -attractors [3, 4], which are a generalization of conformal attractors [32, 33], are described by the Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-g}} = R - \frac{1}{2} K(\partial\phi)^2 - V(\phi) \quad (44)$$

with

$$K = \frac{\alpha}{(1 - \phi^2/6)^2}, \quad V = \alpha f^2(\phi/\sqrt{6}). \quad (45)$$

The Starobinsky model [34] also belongs to this class for a particular choice of f and $\alpha = 1$. Through the change of variable

$$\frac{\phi}{\sqrt{6}} = \frac{1 - \rho}{1 + \rho}, \quad (46)$$

the inflaton Lagrangian becomes

$$\frac{\mathcal{L}}{\sqrt{-g}} = R - \frac{1}{2} \left(\frac{3\alpha}{2\rho^2} \right) (\partial\rho)^2 - \alpha f^2 \left(\frac{1 - \rho}{1 + \rho} \right). \quad (47)$$

From (18) it then follows that for quite generic¹³ f the tree level results are

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12\alpha}{N^2}. \quad (48)$$

As we will motivate in section III D the RG time can be chosen as in (32). The analysis including quantum corrections is then a special case of the general discussion in section II B. As was shown there, the inflationary observables are not affected by quantum corrections as long as F does not break the perturbative expansion in powers of ρ .

C. ξ -attractors

The ξ -attractors are models in which the inflaton is non-minimally coupled to the gravity sector [5, 6].

They are described by a Lagrangian of the form

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{\Omega}{2} R - \frac{1}{2} K_J(\partial\phi)^2 - V_J(\phi). \quad (49)$$

After the usual conformal transformation of the metric $g = \Omega^{-1} g_E$, the gravity sector is in the standard form and the Einstein frame field metric and potential are

$$K = \frac{K_J}{\Omega} + \frac{3}{2} \frac{\Omega'^2}{\Omega^2}, \quad V = \frac{V_J}{\Omega^2}. \quad (50)$$

Let us briefly review which classes of models belong to the ξ -attractor family. In [1] it has been shown that for the special choice $K_J = \frac{1}{4\xi} \frac{\Omega'^2}{\Omega}$, $V_J = \Omega^2 U(\Omega)$ (*special attractors*) the models are completely equivalent to the α -attractors with the identification $1 + \frac{1}{6\xi} \equiv \alpha$. In fact, with this choice of K_J in (50), the Einstein frame field space metric K becomes exactly the one of (47). Other subclasses of ξ -attractors are the *induced inflation models* [6] described by

$$\Omega = \xi f(\phi), \quad K_J = 1, \quad V_J = V_0(\Omega - 1)^2, \quad (51)$$

and the condition that $\Omega \rightarrow 0$ as $\phi \rightarrow 0$; the *universal attractors* [5] satisfy $\Omega \rightarrow 1$ as $\phi \rightarrow 0$, with

$$\Omega = 1 + \xi f(\phi) \quad (52)$$

and the same V_J and K_J as the induced inflation models. Higgs inflation is a particular example of a universal attractor model. For $\xi \rightarrow \infty$ all these models give classically the same predictions at leading order in N^{-1} , which coincide with the predictions of the α -attractors for $\alpha \rightarrow 1$. Indeed, in this limit the first term in K in (50) can be neglected. Thus the Lagrangian in the Einstein frame, after the field redefinition $\rho = \Omega^{-1}$, becomes

$$\frac{\mathcal{L}}{\sqrt{-g}} \simeq \frac{R}{2} - \frac{1}{2} \left(\frac{3}{2\rho^2} \right) (\partial\rho)^2 - V_0(1 - \rho)^2. \quad (53)$$

This is of the form (12)-(13) with the leading pole of order two in the kinetic term ($p = 2$). Therefore the predictions coincide at first order with (48) in the $\alpha \rightarrow 1$ limit and given (23) the conclusions on the RG flow corrections are the same as in the previous sections. In the next section we further discuss the choice of renormalization scale.

D. Renormalization scale for Cosmological Attractors

Whereas the choice of \tilde{t} in Higgs inflation is determined by the known couplings of the Higgs to the Standard Model fields, it is not clear a priori whether \tilde{t} for

¹³ The function f should not be singular at $\phi = 1$, or equivalently at $\rho = 0$, such that it is possible to expand it as $f = V_0(1 + c\rho + O(\rho^2))$.

a generic α or ξ attractor satisfies (23). This depends on the inflaton couplings to the other fields, which is model dependent. Let us consider first the case where only the inflaton loops are relevant in the inflationary regime (which happens when the coupling to all other fields is suppressed).¹⁴ The one loop correction in the Einstein frame is then of the form (remember that the canonical inflaton field is labeled with χ) [36–38]

$$V^{(1)} \propto m_\chi^4 \ln \left(\frac{m_\chi^2}{\mu^2} \right) \simeq m_\chi^4 \ln \left(\frac{H^2}{\mu^2} \right) \quad (54)$$

with the inflaton mass given by¹⁵

$$\begin{aligned} m_\chi^2 &\simeq V_{\chi\chi} - 2H^2 \simeq V \left(\eta - \frac{2}{3} \right) \\ &\simeq -\frac{2}{3} V_0 \left(1 + c \left(1 - \frac{3}{2a_p} \right) \rho + O(\rho^2) \right). \end{aligned} \quad (55)$$

Thus the RG time satisfying (9) is

$$\tilde{t} \simeq \ln(\tilde{m}_\chi^2(\tilde{t})) \sim \ln(H^2) \sim \ln V, \quad (56)$$

where with $\ln(\cdot) \sim \ln V$ we mean that the arguments of the two logs contain the same powers of ρ . This is enough to ensure that (23) is satisfied. The reason for this is simple. If the theory is renormalizable in the EFT sense, the one loop term (as any other log term in the effective potential) can be reabsorbed order by order in the tree level part.¹⁶

Let us now consider the case where the loop corrections are dominated by other fields running in the loops. We focus on the inflaton ϕ coupled to a scalar field σ ; the results straightforwardly generalize to fermion and gauge fields. Here ϕ is the original field appearing in the Lagrangians that define the models, (44) for the α -attractors and (49) for the ξ -attractors. In the small field regime $K \simeq 1$ and $\phi \simeq \chi$, and the ϕ field is the canonical renormalizable field. If we demand the theory to be renormalizable in the small field regime, the coupling between σ and ϕ has to be in a standard renormalizable form. This automatically implies that a one loop term like (56), with

$m_\sigma^2 = \partial^2 V(\phi, \sigma)/\partial \sigma^2$, can be reabsorbed in the tree level potential in the inflationary regime. Indeed, m_σ^4 and V share the same field dependence over the whole field regime. Thus, since the renormalization scale accommodates a subset of the powers of ϕ contained in the classical potential it implies that, once rewritten in terms of ρ , it will equally have a subset of the powers of ρ that are contained in V .

Let us illustrate the previous statements with a simple example. Consider a coupling $\supset g^2 \phi^2 \sigma^2$ in the action (44) [35]. Assume for simplicity that the scalar field σ has no bare mass term. In the small field regime the 1-loop contribution will be of the form $V^{(1)} \sim g^4 \phi^4 \ln(g^2 \phi^2/\mu^2)$, which can be absorbed in a quartic tree level potential $V \sim \lambda \phi^4$. For α -attractors, in the large field regime $V^{(1)}$ and V have exactly the same functional form, which implies $\tilde{t} = 1/4 \ln(\bar{g}^4 \phi^4(\rho))$. This written in terms of ρ using (46) will give a RG time \tilde{t} satisfying (23) (in particular, it will be of the form (32)). A similar applies to the ξ -attractors. The coupling generates loop contributions that can be reabsorbed in the Jordan frame potential V_J over the whole field range, fully analogously to the situation in the α -models. In the large field regime, once we transform to the Einstein frame, all the mass scales (including the renormalization scale, see [2, 29] and [39]) are rescaled as $m \rightarrow m/\Omega^{\frac{1}{2}}$. This still ensures that \tilde{t} is of the form (23); the explicit example in this context is given by Higgs inflation in section III A.

The argument presented so far comes with a caveat. We demanded the low energy regime to be renormalizable, which is not necessary for a working inflationary model. On top, we can relax the constraint by only asking the theory to be renormalizable in the EFT sense in the small field regime; this opens the possibility that the theory is only defined up to some cutoff scale Λ in this regime. To be specific, consider the α/ξ -attractors actions (44) and (49) augmented with an interaction term of the form

$$\mathcal{L}_I = \Lambda^2 g \sigma^2 \ln \left(1 + \frac{\phi^2}{\Lambda^2} \right). \quad (57)$$

Expanding in $\phi \ll \Lambda$ gives the first term of the previous example plus a tower of higher order operators suppressed by powers of the cutoff. The theory is clearly renormalizable in the EFT sense in the small field regime. However, things might change in the large field regime.

In the α models, using the expression of ϕ in terms of ρ given by (46), we get

$$\mathcal{L}_I \sim g \sigma^2 (a_0 + a_1 \rho + O(\rho^2)), \quad (58)$$

where a_i are just the numerical coefficients of the ex-

¹⁴ The general argument in this context [35] states that the one loop correction is negligible since it is suppressed by an extra factor m_χ^2 as compared to the tree level potential.

¹⁵ In all these kind of models we can neglect the backreaction of gravity since $\epsilon \ll \eta$ [29].

¹⁶ The slow roll parameter η in terms of the canonical field is $\eta \propto e^{-\chi/a_2^{\frac{1}{2}}}$. The fact that we can reabsorb the η -dependent term in (55) is basically another way of saying that radiative corrections do not spoil the quasi shift symmetry of the classical potential.

pansion. Therefore, even in the high field regime, the quantum corrections generated by the field σ (proportional to m_σ^4) can be absorbed order by order in the tree level potential. The RG time \tilde{t} satisfies a relation like (56) and the main conclusions of section II B still follow. The same argument does not apply to the ξ -attractors though. Here the interaction term in the Einstein frame becomes

$$\mathcal{L}_I \propto \frac{g\sigma^2}{\Omega^2} \ln \left(1 + \frac{\phi^2}{\Lambda^2} \right) \simeq g\sigma^2 \rho^2 \ln(\rho^{-1}), \quad (59)$$

where for simplicity we have taken $f(\phi) \propto \phi^2$ in (52).¹⁷ This cannot be expanded in powers of ρ as before, leading apparently¹⁸ to a choice for the renormalization scale which does not fulfill condition (23). The interaction renders the theory not renormalizable in the EFT sense. Hence, interaction terms like the one considered here are simply not allowed on this ground — it is important to remember that our whole analysis in section I A is based on the assumption that inflation is described by a perturbatively normalizable EFT.

IV. CONCLUSIONS

In this note we analysed the Cosmological Attractor models using the RG improved action to include the leading log quantum corrections. A consequence of this is that the slow roll parameters and the number of e-folds will depend on the beta-functions, and differ from the respective tree-level expressions. However, when calculating the spectral index and tensor-to-scalar ratio, all corrections exactly cancel. This can be shown, expanding in the small parameter defying the inflationary regime, without explicitly solving the RGEs. This is our main result, the inflationary predictions for α and ξ attractors are not affected by quantum corrections (to leading order in the $1/N$ expansion).

This extends our previous work on Higgs inflation [2] to the larger class of Cosmological Attractors. There is one caveat which allows quantum corrections to become important. It may be that the potential develops an extremum because of the running; this is purely a quantum effect since the tree level potential has no extremum in the inflationary regime. This coincides with the breakdown of our perturbative expansion and analytical control is lost. This allows for hill-top or inflection point inflation which are sensitive to the loop corrections. However, these cases are realized for very fine-tuned values of the parameters and they can be studied numerically for specific inflationary scenarios.

The conclusions remain valid as long as the kinetic term and the potential can be written in the form (13) and the leading log in the effective potential is minimized by an RG function satisfying (23). This turned out to be correct provided the theory is perturbatively renormalizable, in both the low/high field regime; which is the *sine qua non* for our full discussion.

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¹⁷ Similar results follow for generic $f(\phi)$ polynomial in ϕ and generic powers of ϕ appearing in the log in (57).

¹⁸ We say “apparently” because if the theory is not renormalizable even perturbatively, all the arguments of section I A do not apply anymore and talking about RG improved action is misleading.

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